Name:

Warm-up #11

Let $f : X \to Y$ be a function. For a subset $V \subset Y$, let $f^{-1}(V)$ denote the preimage of V defined by

$$f^{-1}(V) = \{x \in X : f(x) \in V\}.$$

(a) Show that $f(f^{-1}(V)) \subset V$. If f is surjective, show that $f(f^{-1}(V)) = V$.

(b) For a subset $U \subset X$, show that $U \subset f^{-1}(f(U))$. If f is injective, show that $U = f^{-1}(f(U))$.