

Name:

Warm-up #19

Let A be a compact subset of the space $C([a, b])$. The goal of this problem is to show that A is uniformly bounded and equicontinuous, thereby establishing a converse to the Arzela-Ascoli theorem.

- (a) For each $\epsilon > 0$, show that there is a finite subset $\{f_1, \dots, f_n\}$ of A such that the collection $\{B_{\epsilon/3}(f_k)\}_{k=1}^n$ covers A .
- (b) Use the fact that each f_k is bounded to show that A is uniformly bounded.
- (c) Use the fact that each f_k is continuous to show that A is equicontinuous.