

**Name:**

Warm-up #20

For a nonnegative integer  $k$ , let  $C^k([a, b])$  denote the space of functions  $f$  on  $[a, b]$  which admit  $k$  *continuous* derivatives  $f', f'', \dots, f^{(k)}$ . Define a norm on  $C^k([a, b])$  by setting

$$\|f\|_{C^k} = \sum_{j=0}^k \sup_{x \in [a, b]} |f^{(j)}(x)|.$$

Use Arzela-Ascoli to show that any sequence  $f_n$  in  $C^k([a, b])$  that is bounded with respect to the  $C^k$ -norm admits a convergent subsequence in  $C^{k-1}([a, b])$ .