Knot Theory Seminar
Problem Set #10
Due Monday, July 20

**1.** Show that a matrix of the form
$$\begin{bmatrix} 0 & \alpha \\ \alpha & \beta \end{bmatrix}$$
for $\alpha, \beta \in \mathbb{R}$ has signature 0.

**2.** Let $A$ be an $n$-by-$n$ symmetric matrix of full rank with sigma series $\{\Delta_n\}$. If $\Delta_{n-1}$ is singular, show that the sigma series is equivalent to one of the form where

$$\Delta'_n = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 & 0 \\ 0 & \lambda_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \alpha \\ 0 & 0 & \cdots & \alpha & \beta \end{bmatrix},$$

where $\alpha, \beta \in \mathbb{R}$ are not both zero, and where $\Delta'_{k-1}$ is formed from $\Delta'_k$ by deleting the last row and the last column of $\Delta'_k$.

**3.** Cromwell 6.9.13.

**4.** Let $P$ be a matrix with integer coefficients. If $P$ has an inverse with integer coefficients, show that $\det P = \pm 1$.

**5.** Show that if $F$ is a Seifert surface spanning a link $L$, then the corresponding Seifert matrix $M$ has $2g(F) + \mu(L) - 1$ rows. (Hint: Problem 10 of PS9.)

**6.** Compute the Alexander polynomials of

(i) The trefoil

(ii) The figure-eight knot

(iii) The Hopf link

(iv) The connected sum of a trefoil and figure-eight.

(v) The disjoint union of two copies of the trefoil.

**7.** Prove Theorem 7.1.4(a) of Cromwell.

**8.** Prove Theorem 7.1.4(e) of Cromwell.

**9.** Prove Theorem 7.1.4(f) of Cromwell.

**10.** For a knot $C$ with genus $g$ and Seifert matrix $M$, show that

$$\det(x(M - M^T)) = x^{2g}.$$