

Knot Theory Seminar
Problem Set #2
Due Monday, June 6

1. Find a polygonal representation of the left-handed trefoil. Can you give a continuous (piecewise) parametrization of this representation? If you are motivated, you can try to apply the reflection $r : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $r(x, y, z) = (-x, -y, -z)$ to this parametrization, and check whether the resulting knot is indeed the right-handed trefoil.
2. Cromwell Exercise 1.2.
3. Cromwell Exercise 1.3.
4. Cromwell Exercise 1.6.
5. The goal of this exercise is to provide an argument for Cromwell Exercise 1.7.
 - (i) Show that if a polygon diagram has at least 3 crossings, then there are at least 6 edges.
 - (ii) Show that if a polygon diagram has at most 2 crossings, then the diagram represents the unknot.
 - (iii) Conclude that there are no knots with polygon index 4 or 5.
6. Cromwell Exercise 1.12.
7. Can you make a diagram for the unknot in any n -by- n grid?
8. How many knot types can you make in a 3-by-3 grid? What about a 4-by-4 grid?
9. Can you find the grid number of the left-handed trefoil knot? Recall that the **grid number** of a knot (type) K is the minimum number n such that K admits a grid diagram of size n -by- n .
10. Can you give a torus representation of the (left-handed) trefoil knot?