Knot Theory Seminar
Problem Set #7
Due Wednesday, July 1

1. Compute the genus of the unknot.

2. Compute the genus of the trefoil knot.

3. Compute the genus of the figure-8 knot.

4. For each positive integer n, construct a knot with genus n.

5. Show that a Seifert graph for a genus 1 surface spanning a knot has rank 2.

6. This exercise explores the notions of "intrinsic" and "extrinsic" properties. Roughly we say that a property of a space is extrinsic if it depends upon some embedding of the space or if it depends upon more data than just the space itself. An intrinsic property is one which depends only on the topological data of the space itself.

(i) The boundary ∂ is an example of an extrinsic property. Recall that for a subset S of a topological space X, we define the boundary of S to be $\partial S = \mathrm{Cl}(S) \setminus \mathrm{Int}(S)$. Let S be the set $[0, 1]$. If we view S as embedded in $\mathbb{R}$, then $\partial S = \{0, 1\}$. Find a space X and an embedding of S into X such that $\partial S = S$. Can you find a space X and an embedding of S into X such that $\partial S = \varnothing$? (Hint: Yes.)

(ii) On the other hand, recall that a subset S of a topological space X is compact if every open cover of S admits a finite subcover. Show that if S is compact, then any embedding of S into another space Y is compact. Hence compactness is an intrinsic property.

(iii) Which of the following are intrinsic and which are extrinsic?

- For a manifold X with boundary, the boundary δX, as defined in Problem Set #5.
- The property of being open.
- The property of being closed.
- The property of being path-connected.
- The property of the complement $X \setminus S$ being path-connected.
- The property of being locally Euclidean of dimension n.
- The property of being Hausdorff.