Goals. The specific research goals of this project are twofold (i) to explore new directions in the representation theory of vertex operator algebras (VOAs) and (ii) to understand further the role of VOAs in quantum field theory (QFT), and specifically, conformal field theory (CFT).

Background. CFT is an attempt at a physical framework of particle interactions that would include all known particles and forces, even gravity. This framework extends the use of Feynman diagrams, whereby one-dimensional diagrams are replaced by two-dimensional world-sheet interactions, which are studied up to conformal equivalence. For genus-zero holomorphic CFT, one corresponding algebraic framework of these interactions is the theory of VOAs [H].

The notion of a VOA has been extended to incorporate both bosons (integral spin particles) and fermions (half-integral spin particles), in addition to supersymmetry of these particles, through the notion of a vertex operator superalgebra (VOSA), which adds the appropriate $\mathbb{Z}_2$-grading to the categorical objects and morphisms [KW]. This extension introduces a new, pivotal automorphism called the parity map, which sends a $\mathbb{Z}_2$-homogeneous element u to $(-1)^{|u|}u$.

If G is a group acting on a VOSA V, then the space V^G of G-invariants enjoys the structure of a VOSA itself. On the geometric side, this amounts to an orbifold theory, whereby the world-sheet manifold is replaced by an orbifold, which is an orbit space M/G of a group G acting on some manifold M. What results is a completely new string theory which is invariant under the action of G. An attempt to understand the modules for V^G is the introduction of a twisted module for V, where the action of V is "twisted" via an automorphism $g \in G$ [B].

Methods. Part (i) of this project will explore this twisted representation theory of vertex operator (super) algebras by studying a super or twisted generalization of a functor, called Zhu's algebra [Z], which is important to the representation theory of VOAs. One such super generalization exists [KW], but this generalization relies heavily on the original formulation of Zhu's algebra, and does not seem to take into account all of the extra "super" structure. Moreover, there is a twisted generalization in the literature [DLM], but I conjecture that this generalization can be extended to an actual functor between appropriate categories and that this functor preserves the monoidal structure of the tensor product. In my senior thesis work, I showed that Zhu's original functor preserves the tensor product. I conjecture that these generalized functors should preserve the tensor product because of this previous work and because of the results of [BDM] and recent results of [BW] and [B], who have related the permutation-twisted modules for a tensor product VO(S)A $V^{\otimes k}$ to the modules for the original VO(S)A V.

Part (ii) of this project will attempt to better understand the role of VOAs in QFT, by exploring the relationships between VOAs and other structures used in the study of QFT, such as factorization algebras [CG], Euclidean topological field theories [ST], and symplectic topology. In particular, Columbia has a strong group of researchers working in problems in symplectic topology and mathematical physics, and I'd like to understand the approaches to quantum field theory in their respective fields. All these approaches offer a different language and setting for quantum field theories, and it would be interesting to know in what ways these dictionaries are compatible and in what ways they diverge.

Activities. I propose the following activities to address these research questions and to set in place a firm foundation for further research related to these ideas.

- Development of a graduate-level research **seminar on approaches to quantum field theory**, with a focus on the role of vertex algebras. Such a seminar will provide me with an avenue to share my explorations and will encourage scheduled research on my proposed questions.

Because attendees of such a seminar will bring their own outside interests, this seminar also offers potential for broader connections and applications of research on vertex algebras and conformal field theory, beyond those outlined above.

- Development of **introductory writings and learning materials** on various topics related to these research questions. I plan to refine my already existing notes and write many more, in order to produce an accessible and wide-ranging series of writings on my areas of research, which I will make available on my website. I eventually plan to collect some of these writings into at least one introductory textbook on a topic in mathematical physics.
- Supervision of **undergraduate reading groups and activities**, designed to expose students to the foundations of research in conformal field theory (such as differential geometry and topology, representation theory, and the theory of vertex algebras), and whenever possible, to immerse the students in research projects of their own. I have experience in this area, as I developed and led two summer research programs for honors math students the past two years.
- Development of a **workshop or summer school** on the role of QFT in modern mathematics. I know that several universities, such as Notre Dame, host workshops run by groups of outside mathematicians, and I would be excited to help lead such a program in the near future.

Broader Impacts. Some broader impacts of this research project include the following.

- **Improved STEM education** and **Enhanced infrastructure for research and education.** This project calls for mentoring undergraduate students, inciting collaboration among graduate students and faculty, and developing course materials and related learning material as well as a route for dissemination through proposed seminars and webpages. Because of the proximity of local schools, this project also permits early exposure of high school students to math research.
- **Improved STEM educator development.** This project will develop the communication skills of graduate students—future educators—through student-run seminars.
- **Increased public scientific literacy.** As an undergraduate, I consistently sought to communicate my research in mathematics with the broader community (see Personal Statement), and I will continue to do so at Columbia. This communication will be achieved not only through public talks and lectures related to this research project, but also through making available learning materials and education related materials on my web page.

[B] K. Barron, Twisted modules for tensor product vertex operator superalgebras and permutation automorphisms of odd order, arXiv:1310.1956.

[BDM] K. Barron, C. Dong, and G. Mason, Twisted sectors for tensor products vertex operator algebras associated to permutation groups, *Comm. Math. Phys.* **227** (2002), 349-384.

[BW] K. Barron and N. V. Werf, Permutation-twisted modules for even order cycles acting on tensor product vertex operator superalgebras, arXiv:1310.3812.

[CG] K. Costello and O. Gwilliam, *Factorization algebras in quantum field theory*, draft, available at http://math.northwestern.edu/ costello/factorization.pdf

[DLM] C. Dong, H. Li, and G. Mason, Twisted representations of vertex operator algebras, *Math. Ann.* **310** (1998) 571-600.

[H] Y.-Z. Huang, *Two-Dimensional Conformal Geometry and Vertex Operator Algebras*, Birkhäuser, Boston, 1997.

[KW] V. Kac and W. Wang, Vertex operator superalgebras and their representations, *Contemporary Mathematics* **175** (1994), 161.

[ST] Supersymmetric field theories and generalized cohomology, *Contemporary Mathematics* **83** (2011), 279.

[Z] Y. Zhu, Modular invariance of characters of vertex operator algebras, *J. Amer. Math. Soc.* **9** (1996), No. 1, 237-302.